Dynamic Modeling and Stabilization of Wheeled Mobile Robot

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Abstract: This paper presents the dynamic modeling of a nonholonomic mobile robot and the dynamic stabilization problem. The dynamic model is based on the kinematic one including nonholonomic constraints. The proposed control strategy allows to solve the control problem using linear controllers and only requires the robot localization coordinates. This strategy was tested by simulation using Matlab-Simulink.

Key-words: Mobile robot, kinematic and dynamic modeling, simulation, point stabilization problem.

1 Introduction

A mobile robot is suitable for a variety of applications in unstructured environments where a high degree of autonomy is required. This desired autonomous or *intelligent* behavior has motivated an intensive research in the last decade. The motion control of nonholonomic wheeled mobile robot (WMR) has been a subject of considerable research over the last few years. Most of the researches are focused on the fact that the WMR does not meet Brockett's well-known necessary smooth feedback stabilization condition [1]. It is recognized that the larger the gap between the controllable and total degrees-of-freedom (DOF) of the WMR, the harder it is to control the robot [2]. Due to this nonholonomic constraint, the WMR cannot be stabilized to a point using the familiar smooth static-state feedback control laws. Therefore, instead of stabilizing the WMR to a point, which at the present stage is still considered as not yet fully generalized, the mobile robot is required to converge to a reference trajectory only. Kanayama *et al*. [3] has first proposed a stable tracking control method for WMR, but it was restricted to the determination of target linear and rotational velocities, involving only kinematic model analysis of WMR. Besides, Yamamoto and Yun have also introduced a look-ahead control algorithm for the mobile platform so that the reference point to be controlled is successful in following the desired trajectory [4]. As inspired by [3], Fierro and Lewis have developed a WMR control scheme through back-stepping the kinematics into the dynamics of WMR with the assumption that a complete prior knowledge of the robotic system is attainable [5]. The others have also provided a global asymptotic control solution for the set point regulation of a general class of nonholonomic systems. Later, Dixon *et al*. [6] suggested a global exponential tracking control method for the stabilization of the nonholonomic WMR. Although these methods are effective, they generally lack the necessary robustness in countering disturbances. The workspace for the WMR is not always ideal and usually packed with various forms of disturbances including frictions, irregular terrains, obstacles in robot’s path, parametric changes and uncertainties within and outside the system, making it almost impossible to model all these disturbances and incorporate them into the dynamics of the WMR. Thus, in order to ensure a more robust and accurate operation of the mobile robot, a disturbance compensation scheme should be incorporated into the operation of the WMR.

The navigation problem may be divided into three basic problems:

- tracking a reference trajectory;
- following a path;
- point stabilization.

Some nonlinear feedback controllers have been proposed in the literature for solving the first problem. The main idea behind these algorithms is to define velocity control inputs that stabilize the closed-loop system. A reference cart generates the trajectory that the mobile robot is supposed to follow. In path following, as in the previous case, we need to design velocity control inputs that stabilize a car-like mobile robot in a given *xy*-geometric path. The most difficult problem is stabilization about a desired posture. Only this problem will be discussed in the paper.

2 Problem Formulation

A simple structure of WMR shown in Fig.1 is a typical example of a nonholonomic mechanical system. It consists in a mobile platform with two differential driving wheels mounted on the same axis and a front free wheel to keep the platform stable. The motion and orientation are achieved by independent actuators of left and right wheels, e.g., DC motors providing the necessary torques to the rear wheels.

2.1 Kinematic Modeling

Let x_c, y_c be the Cartesian coordinates of the point C in the middle of the rear axle respectively x_g, y_g the coordinates of the center of mass of the platform, the point G, and let θ be the angle between the heading direction and the OX-axis specifying the orientation of the platform with respect to the inertial frame. Either the vector of generalized coordinates $\mathbf{q}_g = [x_g, y_g, \theta]^T$ or $\mathbf{q}_c = [x_c, y_c, \theta]^T$ completely specifies the position of the robot in the XOY inertial Cartesian frame. When the same angular velocity is applied to both rear wheels $\omega_r = \omega_l$, the robot can only move in the direction normal to the axis of the driving wheels with linear velocity $\mathbf{v}_c = v \cdot e^{j\theta}$. When the angular velocities applied to the rear wheels are keeping to $\omega_r = -\omega_l$ the robot only turns with angular velocity $\omega = \dot{\theta}$. In the general case, the angular velocities applied to the right and left wheel are different and therefore the robot motion in the horizontal plane is compound. Next relationship is true as long as the mobile satisfies the conditions of *pure rolling and non- slipping:*

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2R} & -\frac{r}{2R} \end{bmatrix} \cdot \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} \quad (1)$$

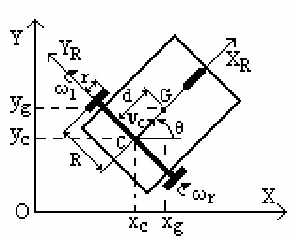

Fig.1 The WMR configuration space

The G point position can be written in vectorial form:

$$\vec{OG} = \vec{OC} + \vec{CG} = \vec{OC} + d \cdot e^{j\theta} \quad (2)$$

The derivatives of the position vectors constitute velocities relationship:

$$\mathbf{v}_g = \mathbf{v}_c + j \cdot d \cdot \omega \cdot e^{j\theta} = (v + j \cdot d \cdot \omega) \cdot e^{j\theta} \quad (3)$$

In the inertial frame the velocity of G point can be expressed in the form:

$$\mathbf{v}_g = \dot{x}_g + j \cdot \dot{y}_g \quad (4)$$

According to (3) and (4), it is possible to express the real and imaginary part of the velocity vector as:

$$\begin{aligned} \dot{x}_g &= v \cdot \cos\theta - d \cdot \omega \cdot \sin\theta \\ \dot{y}_g &= v \cdot \sin\theta + d \cdot \omega \cdot \cos\theta \end{aligned} \quad (5)$$

Eliminating v in the relations (5) results the nonholonomic constraint:

$$\dot{x}_g \cdot \sin\theta - \dot{y}_g \cdot \cos\theta + d \cdot \dot{\theta} = 0 \quad (6)$$

Relation (6) provides that the robot can only move in the direction normal to the axis of the driving wheels as long as the mobile satisfies the conditions of pure rolling and non-slipping.

When the center of mass of the platform, the point G, coincides with its center of rotation, the point C, then $d = 0$ and the relation (6) describes the C point nonholonomic constraint. Defining $\mathbf{S}(\mathbf{q}_g)$ as in (7), the Jacobian matrix that transforms velocities in mobile base coordinates $\mathbf{v} = [v \ \omega]^T$ to velocities in Cartesian coordinates $\dot{\mathbf{q}}_g = [\dot{x}_g \ \dot{y}_g \ \dot{\theta}]^T$, the kinematics or *steering system* of the mobile robot is represented by equation (8) written in matrix form.

$$\mathbf{S}(\mathbf{q}_g) = \begin{bmatrix} \cos\theta & -d \cdot \sin\theta \\ \sin\theta & d \cdot \cos\theta \\ 0 & 1 \end{bmatrix} \quad (7)$$

$$\dot{\mathbf{q}}_g = \mathbf{S}(\mathbf{q}_g) \cdot \mathbf{v} \quad (8)$$

According to (1) and (8) the kinematic model of WMR can be written in the explicitly form:

$$\begin{bmatrix} \dot{x}_g \\ \dot{y}_g \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\cos\theta - \frac{rd}{2R}\sin\theta & \frac{r}{2}\cos\theta + \frac{rd}{2R}\sin\theta \\ \frac{r}{2}\sin\theta + \frac{rd}{2R}\cos\theta & \frac{r}{2}\sin\theta - \frac{rd}{2R}\cos\theta \\ \frac{r}{2R} & -\frac{r}{2R} \end{bmatrix} \cdot \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} \quad (9)$$

It is easy to observe that the robot motion has three degrees-of-freedom (3DOF) while the existing number of controllable degrees-of-freedom is only 2DOF

2.2 Dynamic Modeling

The acceleration of the center of mass $\mathbf{a}_g$ is obtained by derivative of the relation (3):

$$\mathbf{a}_g = (\dot{v} - d \cdot \dot{\theta}^2) \cdot e^{j\theta} + j \cdot (d \cdot \ddot{\theta} + v \cdot \dot{\theta}) \cdot e^{j\theta} \quad (10)$$

The first term is radial component having the same direction as the displacement vector and the second is tangential component. The forward movement of the WMR is produced by the dynamic force $\mathbf{F}_d$ and the

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rotational movement by the dynamic torque $\boldsymbol{\tau}_d$. Denoting m- the total mass of de WMR and I_p- the moment of inertia calculated for rotations around the center of mass, the magnitude of these forces is:

$$\begin{aligned} F_d &= m \cdot \dot{v} - m \cdot d \cdot \dot{\theta}^2 \\ \tau_d &= (I_p + m \cdot d^2) \cdot \ddot{\theta} + m \cdot d \cdot v \cdot \dot{\theta} \end{aligned} \tag{11}$$

On the other hand these forces are generated by the dynamic torques of the two motors, τ_{dr} respectively τ_{dl}:

$$\begin{aligned} F_d &= \frac{1}{r} \cdot (\tau_{dr} + \tau_{dl}) \\ \tau_d &= \frac{R}{r} \cdot (\tau_{dr} - \tau_{dl}) \end{aligned} \tag{12}$$

Taking into account the relations (11) and (12) the dynamic model of WMR is represented by the matrix form:

$$\mathbf{M} \cdot \dot{\mathbf{v}} + \mathbf{C}(\mathbf{v}) = \mathbf{B} \cdot \boldsymbol{\tau} \tag{13}$$

where:

$\mathbf{M} = \begin{bmatrix} m & 0 \\ 0 & I_p + m \cdot d^2 \end{bmatrix}$ – matrix of inertia, symmetric and positive definite

$\mathbf{C}(\mathbf{v}) = \begin{bmatrix} -m \cdot d \cdot \dot{\theta}^2 \\ m \cdot d \cdot v \cdot \dot{\theta} \end{bmatrix}$ - centripetal and Coriolis matrix

$\mathbf{B} = \begin{bmatrix} 1/r & 1/r \\ R/r & -R/r \end{bmatrix}$ - input transformation matrix

$\boldsymbol{\tau} = [\tau_{dr} \;\; \tau_{dl}]^T$ - input dynamic torques matrix

The Lagrange formalism can be also used to find the dynamic equations of the mobile robot. The dynamic model (13) is expressed in the coordinates of the mobile base being possible the narrowing down to two the number of controlled variables. It has to be completed with the dynamic models of the actuating motors. Denoting I_m -the moment of inertia of each wheel and the motor rotor about the wheel axis, τ_{mr}, τ_{ml} -the motor torque of right respectively left motor and τ_{fr} τ_{fr} - the friction torques, the motion equations can be written as:

$$I_m \dot{\omega}_k + \tau_{dk} = \tau_{mk} - \tau_{fk} \quad , \quad \text{with } k = r, l \tag{14}$$

The motor torques τ_{mr}, τ_{ml} are the controlling torques of the robot motion in the Cartesian system. Taking into account the inversed relation (1) and the relation (13) the motion equation (14) can be written as:

$$\mathbf{M}_r \cdot \dot{\mathbf{v}} + \mathbf{C}(\mathbf{v}) + \mathbf{B} \cdot \boldsymbol{\tau}_f = \mathbf{B} \cdot \boldsymbol{\tau}_m \tag{15}$$

where:

$\mathbf{M}_r = \begin{bmatrix} m + \frac{2}{r^2} \cdot I_m & 0 \\ 0 & I_p + \frac{2R^2}{r^2} \cdot I_m + m \cdot d^2 \end{bmatrix}$ -reduced matrix of inertia, symmetric and positive definite;

$\boldsymbol{\tau}_f = [\tau_{fr} \;\; \tau_{fl}]^T$ - friction torques matrix;

$\boldsymbol{\tau}_m = [\tau_{mr} \;\; \tau_{ml}]^T$ - controlling torques matrix.

The relation (15) constitutes the dynamic model of WMR including also the actuators dynamic, represented by the matrix form:

2.3 Stabilization Problem Formulation

The robot stabilization problem can be divided into two different control problems:

- robot positioning control;
- robot orientating control.

The robot positioning control must assure the achievement of a desired position (x_d, y_d), regardless of the robot orientation. The robot orientating control must assure the achievement of the desired position and orientation (x_d, y_d, θ_d).

3 Stabilization Problem Solve

Feedback stabilization consists in finding feedback laws such that an equilibrium point of the closed loop system is asymptotically stable. Unfortunately, the linearization of nonholonomic systems around any equilibrium point is not asymptotically stabilizable. Moreover, there exists *no smooth static (dynamic) time-invariant state-feedback* that makes an equilibrium point of the closed-loop system locally asymptotically.

Fig. 2 shows the stabilization problem, where Δl is the distance between the robot and the desired position (x_d, y_d) in the Cartesian space and θ_d denotes the desired orientation.

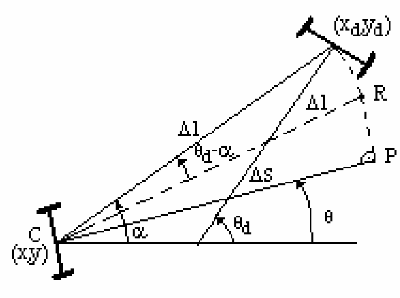

Fig. 2 Robot orientating schema

The robot will not go straight to the final position with a fixed orientation. First, let us define the desired point position related to the point C:

$$\Delta l = \sqrt{(x_d - x)^2 + (y_d - y)^2} = \sqrt{\Delta x^2 + \Delta y^2} \tag{16}$$

$$\alpha = \tan^{-1} \frac{\Delta y}{\Delta x} \tag{17}$$

The proposed control strategy is based on a moving false target by considering the point R ahead of the point C. The point R is relocated at the same distance

Δl related to the point C and is clockwise rotated by the angle $\theta_d - \alpha$ related to the final position. The robot to point R distance is measured in the θ alignment, respectively:

$$\Delta s = \Delta l \cdot \cos(2\alpha - \theta_d - \theta) \quad (18)$$

Now it's possible to make the connection between the dynamic model (15) and the variables s respectively θ.

$$\mathbf{v} = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \dot{s} \\ \dot{\theta} \end{bmatrix} \quad (19)$$

This relation can be integrated, neglecting the integration constant and taking into account the relation (1) results:

$$\begin{bmatrix} s \\ \theta \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2R} & -\frac{r}{2R} \end{bmatrix} \cdot \begin{bmatrix} \theta_r \\ \theta_l \end{bmatrix} \quad (20)$$

where, θ_r and θ_l denote right and left wheel positions. Relation (20) can be used to compute the robot path and orientation at least in simulations.

3.1 Robot Orientating Control

Considering the point R as moving reference point seems normally to denote the angle $\theta_R = 2 \cdot \alpha - \theta_d$ as the reference angle for the orientating control system. In this case the orientating error can be defined as:

$$e_\theta = \theta_R - \theta = 2\alpha - \theta_d - \theta \quad (21)$$

The orientating controller has to assure the convergence of this error to zero for the orientating control problem to be solved. A linear PI controller is used to robot orientating control:

$$u_\theta(t) = K_{p\theta} \cdot e_\theta(t) + K_{I\theta} \cdot \int_0^t e_\theta(\tau) \cdot d\tau \quad (22)$$

The tuning parameters are setting:

$$K_{p\theta} = 2,\ K_{I\theta} = 0.1.$$

3.2 Robot Positioning Control

The robot positioning control problem will be solved by fulfilling the condition: $\Delta s \to 0$. This condition also implies $\Delta l \to 0$ (18), and thereby the robot will be positioned to the desired position. The position error can be accepted as being:

$$e_s = \Delta s \quad (23)$$

where Δs is calculated with (18).

This solution was adopted because the s output signal (the robot path) cannot be measured and in addition is difficult to calculate a suitable value for s_{ref}. Choosing a PI controller:

$$u_s(t) = K_{ps} \cdot e_s(t) + K_{Is} \cdot \int_0^t e_s(\tau) \cdot d\tau \quad (24)$$

having the tuning parameters $K_{ps} = 1,\ K_{Is} = 0.25$, the positioning problem has a reasonable solution. The positioning loop response has to be faster than the orientating loop response to allow an asymptotical stabilization at desired position. The adopted block diagram for the robot asymptotical stabilization is shown in Fig. 3.

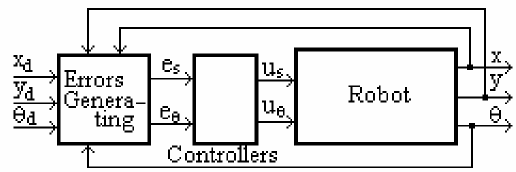

Fig. 3 Robot Control System

The control system is assumed to be equipped with a global positioning system that measures the Cartesian coordinates and the orientating angle. The errors generator uses the relation (18) to compute the position error and relations (17) and (20) to estimate the orientating errors. There are two independent linear controllers to control the linear and angular movement. The inversed relation (20) is used to generate the position references for the two position control systems of the robot wheels. The command vector $[u_s, u_\theta]^T$ substitutes the vector $[s, \theta]^T$ in inversed relation (20) to generate the two reference inputs $\theta_{r\,ref}$, $\theta_{l\,ref}$ of the two robot wheels position control systems.

4 Simulated Results

The proposed control strategy is tested by simulation using Matlab-Simulink environment. The dynamic model (15) was used to build the robot model. The model parameters take for the next values: M = 5 Kg, r = 0.1 m, R = 0.2 m, d = 0.25 m, J_m = 0.002 Kg·m^2, J_p = 0.005 Kg·m^2 and the ratio of the motors motion reduction k_t = 0.1.

In Fig. 4 are shown several robot paths starting at initial conditions $x = 0, y = 0, \theta = 0$ and ending at the desired position $x_d = 1, y_d = 1$ with various orientating angles. The larger desired orientating angle is the longer and more dotted about the robot path is. This seems not to be an advantage but in many applications may be admissible.

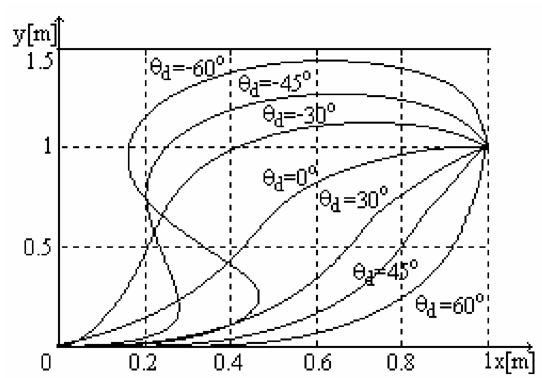

Fig. 4 The robot paths of several desired orientating angles, $x_d=1$, $y_d=1$

The robot paths portrait starting at various initial conditions is shown in Fig. 5. The paths starting at $x>0$ positive coordinate have initial angle set at 180^o and end at the same value of the orientating angle.

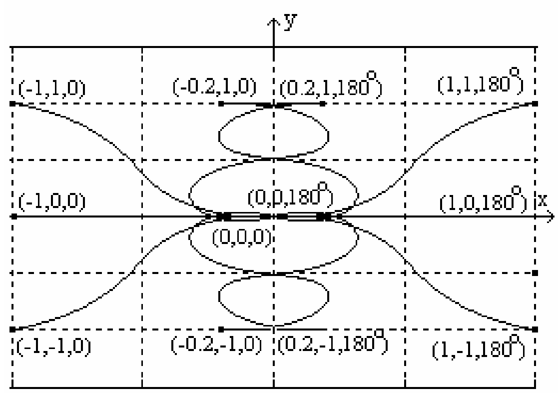

Fig. 5 The robot followed paths starting at various initial conditions (x,y,θ)

When the initial orientating angle is set to zero and the final desired orientating is zero the robot paths are presented in Fig. 6.

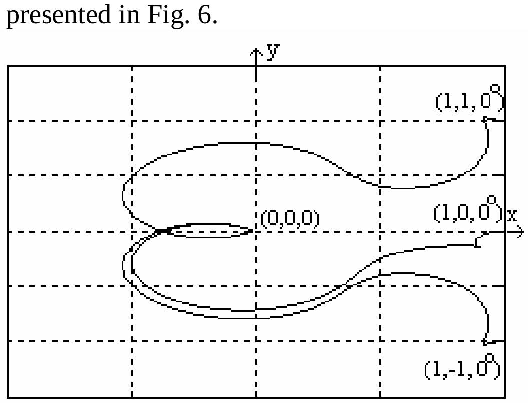

Fig. 6 The robot paths starting with zero orientating angle and x>0

At the beginning of the paths the robot goes back and then it rotates and goes forward on the desired position. For the robot path, starting at (0,0,0) and ending at (1,1,120^o) shown in Fig. 7, the right and left robot wheel angular velocities are presented in Fig. 8 and the position and orientating spacial errors are presented in Fig. 9.

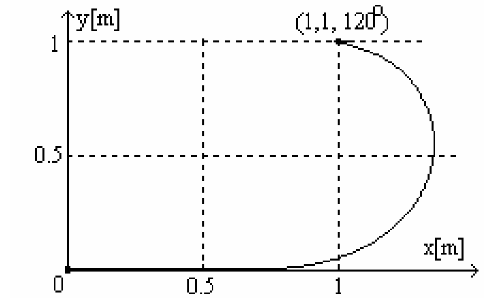

Fig. 7 The robot path starting at (0,0,0) and ending at (1,1,120°)

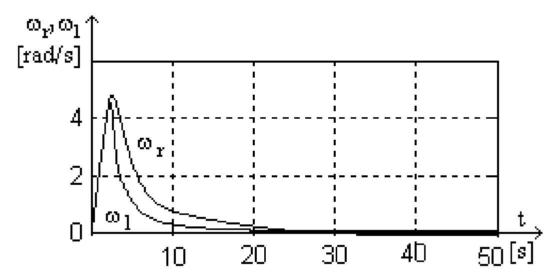

Fig. 8 The right and left robot wheel angular speed

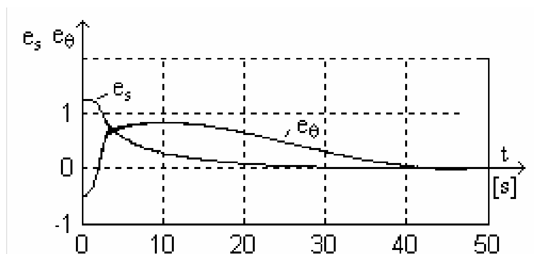

Fig. 9 The position and orientating spacial errors

The errors convergence to zero is visibly in Fig. 9 and fulfills the robot asymptotical stabilization on the desired position.

5 Conclusions

This paper proposes a strategy to solve the asymptotical stabilization problem of a wheeled mobile robot. The main advantage consists in its simplicity. The PI linear controllers are used to control the robot motion from an initial point to final one. The simulated results are satisfactory although the robot path shape can't be forecasted, it mainly depends on the tuning parameters of the controllers and the robot velocities along of the path are not controlled. The velocities control is essential only in following an imposed path.

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